TRINITY COLLEGE

Mathematics Specialist Units 3 & 4
Test 5 2016

Section 1 Calculator Free

Integration: Partial Fractions, Area, Volume, Numerical Differentiation: Implicit, Parametric, Logarithmic

STUDENT'S NAME: _____

DATE: Friday 29th July **TIME:** 25 minutes **MARKS**: 27

INSTRUCTIONS:

Standard Items: Pens, pencils, pencil sharper, eraser, correction fluid/tape, ruler, highlighters, Formula Sheet.

Questions or parts of questions worth more than 2 marks require working to be shown to receive full marks.

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1. (7 marks)

Evaluate the following:

(a) $\int_1^3 \frac{2e^x}{e^x - 1}\,dx$ [3]

(b) $\int_0^{\frac{\pi}{3}} \frac{\sin(x)}{\cos(x)}\,dx$ [4]

2. (7 marks)

Determine $\int \frac{x-4}{x^2-5x+6}\,dx$

3. (6 marks)

Calculate the area trapped between the curves: $y = x, y = \frac{1}{x}$ and the lines $x = \frac{1}{2}$ and $x = 2$.

4. (7 marks)

Given the function $y = x\sin x$, differentiate by:

(a) Using the *Product Rule* [2]

(b) First taking the *Natural Logarithm* of both sides [5]

End of Questions

TRINITY COLLEGE

Mathematics Specialist Units 3 & 4
Test 5 2016

Section 2 Calculator Assumed

Integration: Partial Fractions, Area, Volume, Numerical Differentiation: Implicit, Parametric, Logarithmic

STUDENT'S NAME: _____

DATE: Friday 29th July **TIME:** 25 minutes **MARKS**: 28

INSTRUCTIONS:

Standard Items: Pens, pencils, pencil sharper, eraser, correction fluid/tape, ruler, highlighters, Formula Sheet retained from Section 1.

Special Items: Drawing instruments, templates, three calculators, notes on one side of a single A4 page (these notes to be handed in with this assessment).

Questions or parts of questions worth more than 2 marks require working to be shown to receive full marks.

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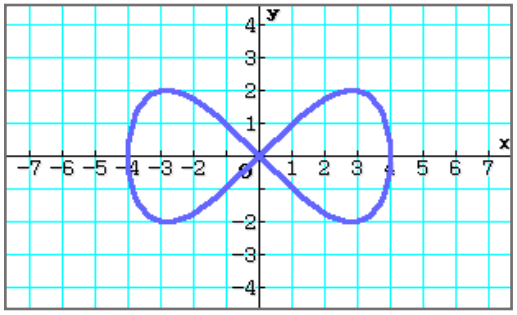
5. (8 marks)



The diagram on the right shows the curve defined parametrically as:

$$x = 4\sin(t),\ y = 2\sin(2t),\ \text{for } 0 \le t \le 2\pi$$

Determine:

(a) an expression for $\frac{dy}{dx}$ in terms of t. [3]

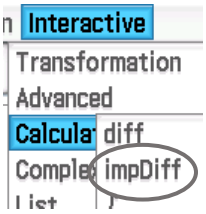
(b) the coordinates and the gradient at the point when $t = \frac{\pi}{6}$. [2]

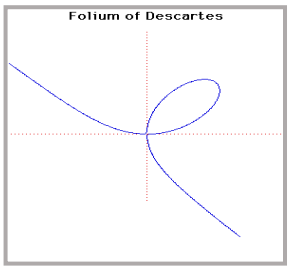
(c) the exact values of t for which $\frac{dy}{dx} = 0$. [3]

6. (11 marks)

The curve $x^3 + y^3 - 9xy = 0$, known as a *folium*, dates back to Descartes in the 1630s.

(a) Use the *implicit differentiation* utility, **impDiff**, on ClassPad to determine $\frac{dy}{dx}$.





[2]

(b) Replicate your result in part (a) by showing all the steps of implicit differentiation. [3]

(c) Determine the equation of the tangent to the curve at the point (2, 4). [2]

(d) Describe the behaviour of the curve by considering $\frac{dy}{dx}$ as x and y tend to $\pm\infty$. [2]

7. (6 marks)

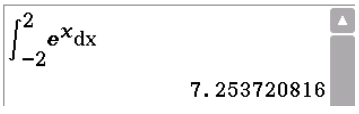
The Numerical Integration *midpoint rule* is that:

$\int_a^b f(x)\,dx \approx w\sum_{i=1}^{n} f\left(\frac{a_{i-1}+a_i}{2}\right)$, where the interval [a, b] is divided into *n* equal width rectangles of width *w* and the values $a_0, a_1, a_2, \ldots, a_n$ are the endpoints of the rectangles, so $a_0 = a$ and $a_n = b$.

(a) Use the midpoint rule to calculate an approximation for $\int_{-2}^{2} e^x dx$ using 8 rectangles.

[5]

(b) Compare your result to this screen capture from ClassPad.



[1]

8. (5 marks)

Calculate the volume of solid generated when the region trapped between the curve: $y = |x(x-1)|$, the x-axis, $x = -1$ and $x = 1$ is rotated about the x-axis.

End of Questions